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# Low-Rank Approximations for Large, Multi-lingual Data

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## Abstract

In this paper we compare low rank approximation methods for data with a particular structure: documents in multiple languages. Rather than looking at only 2 languages in time, we examine the structure in up to 21 languages. The algorithms we choose to compare are  $k$ -means, cross-lingual latent semantic indexing (CL-LSI), and multi-view canonical correlation analysis (mCCA). We test these methods on the European Parliament Proceedings Parallel Corpus.

## 1 Introduction

When extracting topics from documents in multiple languages, we would like to find topics that are not only important within a language but have an corresponding and equivalent representation in the other languages. This could be considered a language-independent measure of importance. Further, by finding a low-rank approximation which is valid over multiple languages, we can use well-established machine learning tools designed for monolingual tasks. Document collections are a typically represented as high dimensional data sets, making them a prime example for lower dimensional representations.

Our work focuses on studying non-probabilistic approaches to multi-lingual dimensionality reduction. The methods we compare are described in section 3. Recently a probabilistic approach based on latent Dirichlet allocation was proposed [1], however no implementations were readily available. We plan to implement it and include it in subsequent studies.

The paper is organized as follows: we first introduce the setting of multilingual data, then describe the algorithms and datasets. We conclude with the evaluation and discussion.

## 2 Multilingual Data

Our data is a collection of documents in multiple languages along with an alignment with correspondences across languages. Individual documents are represented by a vector  $d$  indexed by the terms of a dictionary (the  $i$ -th element is the term frequency ( $TF$ ) in the document). For a corpus, we gather document vectors into the term-document matrix  $D$ . We choose to index the columns by documents and the rows by terms  $D = (d_1, \dots, d_m)$ . Each document is additionally identified by its corresponding language. Each language has its own dictionary independent of other languages.

Primarily, we deal with similarities between documents. Since each language has an independent dictionary, we only define similarity within a language. We note that given a transformation function between dictionaries, we could compute similarities between documents in different languages. However, as translations of individual words is not tailored to the corpus in question and is often not well defined (we often have synonyms, approximate equivalents, etc.), we will use document

correspondences to compute document similarities across languages. Note that this implies a correspondence between columns in each  $D_k$  for all  $k$ . Translations between dictionaries would give us row-based correspondences, but we will investigate this dimension in future work.

Terms in the dictionary are generally not equally important in determining similarity between documents, so we must preprocess the documents. We first use term frequency ( $TF$ ), to prune away infrequent terms. Rather than take a fixed number of top terms in each document we use an adaptive measure. Let  $f(n)$  be a map which returns numbers of terms appearing at least  $n$  times. We find  $n$  such that

$$\frac{|f(n+1) - f(n)|}{\text{original \# of terms}} < 0.001 \quad (1)$$

and used these as the terms for the document. Once this pruning step is complete, we further re-weight the remaining. A term weight should correspond to the importance of the term for the given corpus. The common weighting scheme is called Term Frequency Inverse Document Frequency (TFIDF) weighting. A Inverse Document Frequency (IDF) weight for dictionary term  $j$  is defined as  $w_j = \log(DF_j)$  where  $DF_j$  is the number of documents from the corpora which contain term  $j$ . A document TFIDF vector is its original vector multiplied element-wise by the weights. The  $j$ -th element of a document vector is given by  $TF_j \log(DF_j)$ . Finally, we re-normalize each vector to have Euclidean norm equal to 1.

### 3 Algorithms

We compute the low rank approximation of the term-document matrix using three algorithms:  $k$ -means[2], cross-lingual latent semantic indexing(CL-LSI)[3], and multi-view canonical correlation analysis (mCCA)[4].

$k$ -means is perhaps the most well-known and used clustering algorithm. To make the data compatible with  $k$ -means, we merge all the term-document matrices into a single matrix by stacking the individual term-document matrices.

$$D_{\text{Total}} = [D_1^T, D_2^T, \dots, D_\ell^T]^T \quad (2)$$

such that the columns respect the alignment of the documents. Therefore, each document is represented by a long vector indexed by the terms in all languages. It is these vectors which determine the similarity on which the  $k$ -means is computed.

The next method is CL-LSI which is a variant of LSI [5] for more than one view. Each view in this context represents a language. It merges the individual term-document matrices in the same way as we did for  $k$ -means. LSI computes a singular value decomposition of  $D_{\text{Total}}$ . Since the matrix can be large we can use an iterative method like the Lanczos [6] algorithm to find the left singular vectors corresponding to the largest singular values.

Finally, we test mCCA is specifically designed to consider data from multiple sources (in this case languages). Therefore, we do not merge the individual term-document matrices into one as in the other two methods. For each language (view), we estimate the pairwise correlation coefficients, then we try to find vectors which maximize the sum of all pairwise correlations over all languages. This can be written as the following optimization

$$\max_{w_1, \dots, w_\ell} \sum_{i=1}^{\ell} \sum_{j=i+1}^{\ell} \frac{w_i^T D_i D_j^T w_j}{\sqrt{w_i^T D_i D_i^T w_i} \sqrt{w_j^T D_j D_j^T w_j}} \quad (3)$$

However, this allows only for a 1-dimensional representation (i.e. one vector per language). Since we would like to allow for more vectors per language, we denote the double sum in Equation 3 by  $\text{mCCA}(w_1, \dots, w_\ell)$ . If  $\text{corr}$  denotes the correlation, to find  $M$  vectors, the optimization objective function becomes

$$\max_{\substack{w_i^{(j)} ; i=1, \dots, \ell \\ j=1, \dots, M}} \sum_{s=1}^M \text{mCCA} \left( w_1^{(s)}, \dots, w_\ell^{(s)} \right), \quad \text{s.t.} \quad \text{corr} \left( w_i^{(s)}, w_i^{(t)} \right) = 0 \quad \forall s \neq t \quad (4)$$

We require that the set of  $M$  vectors we return for each language are uncorrelated (so that we do not get copies of a vector) which together maximize the pairwise correlation between languages.

## 4 Data Set

To investigate the empirical performance of the low rank approximations we will test the algorithms on a large-scale, real-world multilingual dataset: the European Parliament Proceedings Parallel Corpus v6 (EuroParl)[7], a corpus released by the EU Parliament. This source offers a large number of comparable documents in multiple languages. In particular, EuroParl (Release v6) provides transcripts of parliamentary session in almost all the EU languages. This can be considered a *gold standard* in terms of multi-lingual data: these documents are professional translations of each other. In the corpus, each day is available as a separate file, although newer data is available in smaller units. To create the document, within each file, we create a document for each speech ID<sup>1</sup>(so each speech is a document). In the end, the document contains a document ID, a speech ID, and paragraph markup. The documents were aligned in two ways: by speaker id and by paragraphs. The number of paragraphs was used only if the number was the same for different languages. In total, it contains approximately 21000 documents in the 21 languages of the EU taking up around 2GB of storage.

## 5 Evaluation

We measure the performance of the low rank approximation using two metrics: mean average precision mate retrieval and correlation between monolingual similarity profiles between the query document and its nearest neighbour in the joint representation. For each evaluation, we randomly select a training set and test set from the data.

The first evaluation criteria we is the *mean average precision mate retrieval score* (AMPMR). This measures the similarity between the documents and their translations in the common vector space induced by the latent model. Good models map the documents close to their translations - indicating that some language independent (semantic) information was captured. We evaluate each latent model (given by projection operators  $P_1$  and  $P_2$ ) by considering a pair of aligned test sets  $T_1$  and  $T_2$  in languages  $L_1$  and  $L_2$ . We select a query document  $q_1 \in T_1$  and denote the corresponding translated document  $q_2 \in T_2$ . We then compute the projections  $P_1q$  and  $P_2T_2$  and rank the elements of  $P_2T_2$  by their similarity to  $P_1q$  in the projection space (measured by Euclidean distance). The mean average precision mate retrieval score is the inverse of the rank of  $P_2q_2$ .

This score does not give us a complete picture. The low rank of a mate document does not necessarily indicate poor performance if the documents which outranked it share similar content to the query. Therefore, we compute an alternative performance measure: *correlation between monolingual similarity profiles* (CMSP). As before, we choose a query document, target test corpus, and project them to a common vector space. From the target corpus, we select the closest document  $r \in L_2$  to the query in the projection space. We then compute two similarity profile vectors:  $v_1$  contains the monolingual cosine similarity between  $q_1$  and all the documents in  $T_1$  and similarly  $v_2$  contains the cosine similarities between  $r$  and  $T_2$ . The score is the correlation coefficient between  $v_1$  and  $v_2$ .

The results for each of the two measures for several fitting parameters are in Tables 1<sup>2</sup>. We obtain the results by averaging over all queries in each test set, all pairs of languages and over ten repetitions of the choice of training and test data. The experiments are indexed by the number of training samples (Ntrain), the number of languages we consider (NViews) and the dimensionality of the latent space(Ndims). We also ran a preliminary experiment on a data set based on Wikipedia<sup>3</sup> for the same languages. The result is in the final row of Table 1.

## 6 Discussion

The results illustrate that both LSI and mCCA outperform  $k$ -means in terms of our evaluation criteria. We believe that this is due to mCCA and LSI capturing the word co-occurrence patterns, a well-established fact for the LSI method, which also holds for mCCA (The decomposition is based

<sup>1</sup>In the corpus, this is referred to as the speaker ID.

<sup>2</sup>Updated results can be found in the full version of this paper [8]

<sup>3</sup><http://www.wikipedia.org>

Table 1: Mean Values for Experiments

Parameters (Ntrain,NViews,Ndims)	AMPMR			CMSP		
	$k$ -means	LSI	mCCA	$k$ -means	LSI	mCCA
(100, 3, 5)	0.1096	0.2306	0.2184	0.1135	0.2007	0.1889
(100, 3, 20)	0.3906	0.5292	0.5151	0.3308	0.4466	0.4329
(100, 10, 5)	0.08166	0.1870	0.2006	0.0833	0.1685	0.1725
(100, 10, 20)	0.3204	0.4546	0.4623	0.2686	0.3901	0.3911
(100, 21, 5)	0.09468	0.2040	0.1868	0.1006	0.1789	0.1657
(100, 21, 20)	0.3255	0.4773	0.4577	0.2730	0.4076	0.3878
(1000, 3, 5)	0.1090	0.4059	0.3572	0.1162	0.3268	0.2837
(1000, 3, 20)	0.4180	0.8518	0.8748	0.3465	0.7725	0.7967
(1000, 10, 5)	0.08320	0.3126	0.3801	0.0894	0.2533	0.2965
(1000, 10, 20)	0.3416	0.7695	0.7954	0.2880	0.6921	0.7148
(1000, 21, 5)	0.08427	0.3276	0.3187	0.0783	0.2618	0.2494
(1000, 21, 20)	0.3702	0.8002	0.8075	0.3110	0.7228	0.7240
(1000, 10, 10)	0.2155	0.2335	0.3169	0.2612	0.2779	0.3362

on the inter-lingual covariance matrix). The performance of LSI and mCCA is comparable in all the tested cases. This is unexpected, since LSI discards the correspondence information between feature (word) and language. Note that the above tables are coarse measures, as we average over all pairs of languages. The two algorithms may have different fail cases which may explain why this correspondence information is does not seem important. Furthermore, in our preliminary experiment, mCCA performed significantly better on the Wikipedia data set. However, additional experimentation is required before we can draw conclusions. Finally, we note that the results show there is still room to improve the methods which leads us to believe that a further investigation into low-rank approximations of multilingual structure is warranted.

## References

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## A Learned Concept Vectors

We include the top 20 vectors we learn, each dimension 5 for illustrative purposes. We only include 4 languages here, see [8] for more examples.

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women item report voted resolution  
you your barroso he mr  
women gender equality violence men  
you young economic crisis strategy  
vote you minutes details item  
vote minutes details results see  
you acp trade agreement european  
euro countries greece acp commission  
young budget treaty parliament council  
acp agricultural budget agreement farmers  
turkey women market accession policy  
presidency trafficking violence council turkey  
trafficking vote agricultural policy farmers  
turkey trafficking balkans accession montenegro  
item israel strategy internal trafficking  
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arctic products greece treaty cosmetic  
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femmes appelle vote rapport l'ordre  
vous avez barroso votre pouvez  
femmes hommes l'egalite violence vous  
vous jeunes crise strategie economique  
votes vote vous verbal commission  
votes verbal vote proces resultats  
vous acp turquie europeen l'accord  
euro grece commission acp zone  
jeunes budget traite parlement lisbonne  
acp budget agricole pac l'accord  
turquie femmes marche interieur politique  
presidence violence turquie conseil etres  
traite vote pac etres agricole  
turquie balkans occidentaux montenegro kosovo  
strategie interieur l'ordre appelle israel  
2020 strategie entreprises bei pme  
turquie acp jeunes interieur presidence  
produits cosmetiques traite zone grece  
russie arctique l'arctique jeunes sante

pisemna clanek rozprava ukoncena prohlaseni  
zen bodem dalsim pisemne usneseni  
jste barroso vam vas mužete  
zen nasili zeny pohlavi zenach  
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dohody akt jste partnerstvi komise  
akt komise eurozony zeme recko  
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akt dohody zemedelske klimatu zemedelstvi  
zen turecko pristoupeni turecka trhu  
predsednictvi nasili obchodovani lidmi rady  
obchodovani hlasovani lidmi pisemne soudrznosti  
makedonie obchodovani turecko lidmi pristoupeni  
bodem strategie obchodovani izrael dalsim  
2020 eib bodem podniky demokratuž  
akt turecko predsednictvi turecka trh  
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rusko bodem dalsim zdravi arktidy

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abstimmung the protokoll siehe abstimmungsstunde  
akp turkei abkommen kommission staaten  
eurozone griechenland akp kommission lander  
lissabon parlaments parlament arbeitsmarkt vertrag  
akp agrarpolitik abkommen versammlung gap  
turkei frauen binnenmarkt agrarpolitik gap  
gewalt menschenhandel turkei ratsvorsitz prasidentschaft  
menschenhandel agrarpolitik abstimmung gap landwirte  
turkei menschenhandel mazedonien montenegro kosovo  
israel binnenmarkt strategie punkt menschenhandel  
2020 strategie eib unternehmen kopenhagen  
turkei akp binnenmarkt gewalt versammlung  
arktis nanomaterialien eurozone griechenland vertrag  
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